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D-branes from N non-BPS D0-branes

Josef Klusoň

Department of Theoretical Physics and Astrophysics
Faculty of Science, Masaryk University
Kotlářská 2, 611 37, Brno, Czech Republic
E-mail: klu@physics.muni.cz

Abstract: In this paper we would like to show that from N non-BPS D0-branes in type-IIB theory we can obtain all BPS and non-BPS D-branes through tachyon condensation in the limit $N \to \infty$.

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Contents

1. Introduction

In the recent years there was a significant progress in the understanding of the unstable configurations in superstring theories. This work has been pioneered with the seminar papers by A. Sen [1]. It was argued in [2, 3] that all D-branes can arise as solitonic solutions in the world-volume theory of the unstable configurations of D-branes. (For review of the relation between K-theory and D-branes, see [4], for recent discussion, see [5, 6, 7].)

Evidence for this proposal was given from the analysis of CFT description of this system [1], for review of this approach, see [8, 9]. It was also shown on many examples that string field theory approach to this problem is very effective one which allows to calculate tachyon potential [10]–[26]. Recently this problem was also studied from the point of view of the Witten's background independent open string field theory [28]–[35]. Success of string field theory in the analysis of tachyon condensation indicates that string field theory could play more fundamental role in the non-perturbative formulation of string theory.

The second approach to the problem of tachyon condensation is based on the non-commutative geometry [36]. This analysis has been inspired with the seminal paper [37]. Application of this approach to the problem of the tachyon condensation was pioneered in [38, 39]. This research was then developed in other papers [6], [41]–[45].

In this paper we would like to discuss the problem of the tachyon condensation from slightly different point of view. We would like to show that non-trivial tachyon condensation is also possible in the action for N non-BPS D0-branes, which results in the emergence of higher-dimensional BPS and non-BPS D-branes in the process similar to the emergence of higher-dimensional branes in Matrix theory [46, 47] (For review, see [48]–[51]). However, there is an important difference. In matrix theory we work with the exact form of the action and with the maximal supersymmetric theory which allows to obtain exact result. On the other hand, we do not know exactly the form of a non-BPS D-brane action which can be guessed only on some general grounds. Possible form of this action was proposed in seminal paper [52], other attempts to define this action appeared in [53]–[58]. We also study system with maximally broken supersymmetry so that the analysis is much more difficult. However, we still believe that our approach is useful since it presents an evidence of the emergence of higher-dimensional D-branes from lower-dimensional ones thanks to the tachyon condensation. As we will see the resulting configurations carry the correct RR charges which allows us to expect that these solutions are well defined.

We start in section 2 with discussion of the bosonic form of the action for N non-BPS D9-branes. Then we use T-duality transformation, following [56], in order to obtain an action for N non-BPS D0-branes.

In section 3 we will discuss possible applications of this action. Since we do not know the exact form of this action, we restrict ourselves only to the linear approximation. We also show that the whole action (without restriction to the linear approximation) contains the solution corresponding to collection of non-BPS D0-branes. We also show that this action has a solution corresponding to the tensionless D0-branes, discussed recently in the case of non-commutative field theories [38, 39].

In sections 4 and 5 we will show that from the collection of N non-BPS D-branes we can obtain all BPS and non-BPS D-branes in the limit $N \to \infty$ when we can replace infinite-dimensional matrices with operators acting on some abstract Hilbert space. We will proceed in the same way as in the study of tachyon condensation in non-commutative theories [38, 39, 42] and we will show that these D-branes carry non-zero RR-charge thanks to the existence of generalised Wess-Zumino term [59].

In the next section we start with the action for non-BPS D0-branes, which can be obtained from the action for non-BPS D9-branes through T-duality transformation.

2. Non-BPS D-brane action

In this section we will discuss the possible form of the action for N non-BPS D-branes, following the seminal paper [52]. Similar discussions were presented in [53]–[57].

We start with the most general form[1] of the action for N non-BPS D9-branes in the form

$$S=-\frac{C_9}{g_s}\int d^{10}x\,\mathrm{Str}\left(\sqrt{-\det(E_{\mu\nu}+\lambda F_{\mu\nu})}\left[\sum_{n=1}^{\infty}f_n(T^2)(\lambda E^{\mu\nu}D_\mu T D_\nu T)^n+V(T)\right]\right), \tag{2.1}$$

where

$$\lambda=2\pi\alpha'\,,\qquad C_p=\sqrt{2}T_p\,,\qquad T_p=\frac{2\pi}{(4\pi^2\alpha')^{(p+1)/2}} \tag{2.2}$$

and

$$\begin{aligned}E_{\mu\nu}&=G_{\mu\nu}+B_{\mu\nu}\,,\qquad D_\mu T=\partial_\mu T+i[A_\mu,T]\,,\\ F_{\mu\nu}&=\partial_\mu A_\nu-\partial_\nu A_\mu+i[A_\mu,A_\nu]\,,\qquad \mu,\nu=0,\dots,9\,.\end{aligned} \tag{2.3}$$

The gauge field A_μ belongs to the adjoint representation of the gauge group $\mathrm{U}(N)$. Finally, $V(T)$ is the potential for the tachyon. We do not know much about functions $f_n(T^2)$ with exception that should be even functions of its argument [52]. There is also one intriguing conjecture [56, 57] which says that these functions could be equal to the tachyon potential and consequently should be equal to zero for $T^2=T^2_{\mathrm{min}}$, where T_{min} is a tachyon value at the local minimum. In this paper we will suppose that these functions do not need directly equal to $V(T)$ but we will assume that they are zero for $T=T_{\mathrm{min}}$ and also that they obey $df_n(T)/dT=0$ for $T=T_{\mathrm{min}}$. In other words, we expect these functions in the form

$$f_n(T^2)=\sum_{m=1}b_{nm}(T^2-T^2_{\mathrm{min}})^m\,. \tag{2.4}$$

Then the kinetic term has a form

$$\sum_{n=1}\sum_{m=1}b_{nm}(T^2-T^2_{\mathrm{min}})^m(\lambda E^{\mu\nu}D_\mu T D_\nu T)^n\,. \tag{2.5}$$

In (2.1) the Str means the symmetrised trace [60] $\mathrm{Str}(A_1,\dots,A_n)=\frac{1}{n!}(\mathrm{Tr}\,A_1\dots A_n$ $+\,\mathrm{permutations})$. In this trace we consider the field strength F and covariant derivative DT as a single object as well as $(T^2-T^2_{\mathrm{min}})$, otherwise we could not obtain the result that the action is equal to zero for $T=T_{\mathrm{min}}$. The prescription for including the symmetrised trace was suggested in [60]. The evidence for this proposal was further given in [61, 62]. We must also stress one important thing. It seams to us that the tachyon potential should appear as a single object in the action (as for example a covariant derivative) in order to obtain from the action the correct value of the tachyon ground state and also in order to obey the requirement that for the tachyon

[1] We mean the most general form up to the first derivatives. Of course, there could be infinite number of higher derivatives. In the case when commutators are small the action with the first derivatives is the appropriate one.

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equal to its vacuum value the action should vanish. When we used the potential as a matrix valued function, than the symmetrised trace would lead to the different arrangements of the various terms from the tachyon potential and we do not know how we could get a sensible result.

In order to obtain the action for lower-dimensional D-brane, we use T-duality in the same manner as in [56, 59]. Let us consider T-duality on a set of directions denoted with $i, j = p+1, \ldots, 9$. The fields transform as [63]

$$\tilde{E}_{ab} = E_{ab} - E_{ai}E^{ij}E_{jb}\,, \qquad \tilde{E}_{aj} = E_{ak}E^{kj}\,, \qquad \tilde{E}_{ij} = E^{ij}\,, \tag{2.6}$$

where $a, b = 0, 1, \ldots, p$ and E^{ij} denotes the inverse of E_{ij}, i.e. $E_{ik}E^{kj} = \delta_i^j$. One also has a dilation transformation

$$e^{2\tilde{\phi}} = e^{2\phi}\det(E^{ij})\,. \tag{2.7}$$

Now T-duality acts to change the dimension of D-brane world-volume. We have two possibilities: If a coordinates transverse to Dp-brane, e.g. $y = x^{p+1}$ is T-dualised, it becomes D(p+1)-brane where y is now extra world-volume dimension. If a coordinate on the world-volume of Dp-brane is T-dualised, e.g. $y = x^p$, it becomes D($p-1$)-brane where y is now extra transverse dimension. In the first case, we have a rule for transformation of the world-volume fields

$$\Phi^{p+1} \Longrightarrow A_{p+1}\,, \tag{2.8}$$

and in the second case

$$A_y \Longrightarrow \Phi^y\,. \tag{2.9}$$

In the second case, the corresponding field strength transforms as

$$F_{ay} \Longrightarrow D_a\Phi^y\,. \tag{2.10}$$

In the T-duality transformation along the world-volume coordinate x^p we presume that all field are independent on this coordinate

$$\partial_p\Psi = 0\,. \tag{2.11}$$

However, this rule does not imply that $D_{x^p}\Psi$ is equal to zero, rather we obtain

$$D_p\Psi \Longrightarrow i[\Phi^p, \Psi]\,. \tag{2.12}$$

Now we are ready to discuss the action for non-BPS Dp-brane. We obtain this action from (2.1) applying T-duality transformations in $p+1, \ldots, 9$ dimensions, following [56, 59]. We will discuss the term

$$\tilde{D} = \det(\tilde{E}_{\mu\nu} + \lambda F_{\mu\nu})\,. \tag{2.13}$$

With using (2.6) we get

$$D = \det \begin{pmatrix} E_{ab} - E_{ak}E^{kj}E_{jb} + \lambda F_{ab} & E_{ak}E^{kj} + \lambda D_a \Phi^j \\ -E^{ik}E_{kb} - \lambda D_b \Phi^i & E^{ij} + i\lambda[\Phi^i, \Phi^j] \end{pmatrix} . \tag{2.14}$$

When we use the mathematical formula

$$\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det \begin{pmatrix} A - BD^{-1}C & B \\ 0 & D \end{pmatrix} = \det(A - BD^{-1}C)\det(D) , \tag{2.15}$$

we obtain [59]

$$D = \det \left(P\left[E_{ab} + E_{ai}(X^{ij} - \delta^{ij})E_{jb}\right]\right) \det(Q^i_m)\det(E^{mj}) , \tag{2.16}$$

where we have defined

$$Q^{ij} = E^{ij} + i\lambda[\Phi^i, \Phi^j] , \qquad P[E_{ab}] = E_{ab} + 2\lambda E_{ai}D_b\Phi^i + \lambda^2 E_{ij}D_a\Phi^i D_b\Phi^j , \tag{2.17}$$

and

$$X^{kl} = E^{ki}(Q)^{-1}_{ij}E^{jl} . \tag{2.18}$$

We have also used

$$\det(Q^{ij}) = \det(Q^{im}E_{mk}E^{kj}) = \det(Q^i_m)\det(E^{mj}) . \tag{2.19}$$

The analysis of F function in (2.1) is straightforward and we get

$$F = V(T) - \sum_{n=1}^{\infty} f_n(T)\lambda^n \left((E^{ab} - E^{ai}E_{ij}E^{jb})D_aTD_bT + 2iE^{ak}E_{kj}[\Phi^j, T]D_aT - \right.$$
$$\left. - E_{ij}[\Phi^i, T][\Phi^j, T]\right)^n .$$

With using (2.7) we obtain the action for non-BPS Dp-brane

$$S = -\frac{C_p}{g_s}\int d^{p+1}\sigma \,\mathrm{Str}\left(\sqrt{-\det(P[E_{ab} + E_{ai}(X^{ij} - \delta^{ij})E_{jb}]} \times \right.$$
$$\left. \times \sqrt{\det Q^i_j} F(T, DT, \ldots)\right) . \tag{2.20}$$

3. Applications

In this section we will discuss the various applications of the previous action. We will work with the non-abelian action for N non-BPS D0-branes in type-IIB theory. Thanks to gauge invariance, we can pose $A_0 = 0$. Than the covariant derivative is $D_t\Phi = \dot{\Phi}$. We will work in the flat space-time background

$$E_{ab} = \eta_{ab} , \qquad a, b = 0 , \qquad E^{ij} = \delta^{ij} , \qquad i, j = 1, \ldots, 9 . \tag{3.1}$$

Then we have

$$\begin{aligned} Q^{ij} &= \delta^{ij} + i\lambda[\Phi^i, \Phi^j]\,, \\ P[E_{ab}] &= -1 + \lambda^2(\dot{\Phi}^i)^2\,, \\ P[E_{ai}X^{ij}E_{jb}] &= \lambda^2\dot{\Phi}^k\delta_{ki}X^{ij}\delta_{jl}\dot{\Phi}^l\,, \end{aligned} \tag{3.2}$$

and finally

$$P[E_{ab} + E_{ai}(X^{ij} - \delta^{ij})E_{jb}] = -1 + \lambda^2(\dot{\Phi}^i)^2 + \lambda^2\dot{\Phi}^k\delta_{ki}(X^{ij} - \delta^{ij})\delta_{jl}\dot{\Phi}^l\,. \tag{3.3}$$

We will work in the leading order in λ in which the previous expression reduces into

$$-1 + \lambda^2(\dot{\Phi}^i)^2 \tag{3.4}$$

and $F(T, \dots)$ in the leading order approximation has a form

$$F = V(T) + f(T)\lambda\dot{T}\dot{T} + \lambda f(T)\delta_{ij}[\Phi^i, T][\Phi^j, T]\,. \tag{3.5}$$

Using these results we obtain the action for N non-BPS D0-branes in the leading order approximation

$$\begin{aligned} S = \frac{C_0}{g_s}\int dt\, \mathrm{Str}\Big(&-V(T) + \frac{\lambda}{2}\dot{\Phi}^i\dot{\Phi}^j\delta_{ij} - \frac{\lambda^2}{4}\delta_{kl}\delta_{mi}[\Phi^i, \Phi^k][\Phi^l, \Phi^m]V(T) + \\ &+ \lambda f(T)\dot{T}\dot{T} + \lambda f(T)\delta_{ij}[\Phi^i, T][\Phi^j, T]\Big)\,. \end{aligned} \tag{3.6}$$

In what follows we will consider static configurations only, when all fields are time independent. Then the action is

$$\begin{aligned} S &= -\frac{C_0}{g_s}\int dt\mathcal{V}(T, \Phi)\,, \\ \mathcal{V}(T, \Phi) &= \mathrm{Str}\left(V(T) + \frac{\lambda^2}{4}[\Phi^i, \Phi^j][\Phi^j, \Phi^i]V(T) - \lambda f(T)[\Phi^i, T][\Phi^i, T]\right)\,. \end{aligned} \tag{3.7}$$

We will also consider the coupling of the non-BPS D-brane to the external RR field. This term was calculated in [64] for single non-BPS D-brane and we have generalised this term for N non-BPS D-branes in [54, 55]. Applying T-duality rules on these terms give complicated expression which was discussed recently in [65]. However in this paper we will discuss only the leading order term which for non-BPS D0-branes has a form

$$\frac{\mu_{-1}}{2T_{\min}}\int dt\, \mathrm{Str}\, P\left[e^{i\lambda\mathbf{i}_\Phi\mathbf{i}_\Phi}\left(\dot{T} + i[\mathbf{i}_\Phi, T]\right)\sum_n C^{(n)}\right]\,, \tag{3.8}$$

where P is a pull-back to the world-volume of D0-brane and $\mathbf{i}_\Phi$ is the interior product [66]. Acting on forms, the interior product is an anti-commuting operator of form degree -1, e.g.

$$\begin{aligned} C^{(2)} &= \frac{1}{2} C^{(2)}_{\mu\nu} dx^{\mu} \wedge dx^{\nu} \,, \\ \mathbf{i}_v C^{(2)} &= v^{\mu} C^{(2)}_{\mu\nu} dx^{\nu} \,, \\ \mathbf{i}_w \mathbf{i}_v &= w^{\nu} v^{\mu} C^{(2)}_{\mu\nu} = -\mathbf{i}_v \mathbf{i}_w C^{(2)} \,. \end{aligned} \tag{3.9}$$

We will see that this Wess-Zumino term (WZ) correctly reproduces the charges of higher-dimensional D-branes arising from tachyon condensation on the system of N non-BPS D0-branes.

We start with simple examples of tachyon condensation which are the solutions of the whole action as well. The first one is [41]

$$T = T_{\min}(1 - P_k) = \mathrm{diag}\left(0, \dots, 0^k, T_{\min}, \dots, T_{\min}^{N-k}\right) , \qquad \Phi^i = 0 \,, \tag{3.10}$$

where P_k is a projector on the fist k states which has the form $P_k = \mathrm{diag}(1, \dots, 1^k)$. It is easy to see that this is a solution of the equation of motion since all commutators vanish (we do not need presume the condition $f_n(T_{\min}) = 0$) and also it is easy to see that $dV/dT = 0$. The energy of this configuration is equal to

$$E = \frac{C_0}{g_s} \mathrm{Str}\, V(T) = \frac{C_0}{g_s} \mathrm{Tr}\, V(T) = \frac{C_0}{g_s} k \,, \tag{3.11}$$

where we have used $V(0) = 1$ [10]. This is the rest energy of k non-BPS D0-branes. However, there is also one non-trivial solution corresponding to tensionless D0-branes [41]

$$T = T_{\min}(1 - 2P_k) = \mathrm{diag}\left(-T_{\min}, \dots, -T^k_{\min}, T_{\min}, \dots, T_{\min}^{N-k}\right) , \qquad \Phi^i = 0 \,. \tag{3.12}$$

As in the previous solution commutators are equal to zero and the variation of the potential gives

$$\frac{\delta V(T)}{\delta T} = \sum_{n=1}^{\infty} n a_n 2T (T^2 - T^2_{\min})^{n-1} \,, \tag{3.13}$$

since we can presume that the potential has a form $V(T) = \sum_{n=1}^{\infty} a_n (T^2 - T^2_{\min})^n$. Then we can immediately see that (3.12) is a solution which is a extreme of the potential since

$$T^2_{\min}(1 - 2P_k)^2 = T^2_{\min} \,. \tag{3.14}$$

The energy of this configuration is equal to

$$E = \frac{C_0}{g_s} \mathrm{Tr}\, V(T) = 0 \,, \tag{3.15}$$

since $T^2 = T^2_{\min} 1_{N\times N}$. What is a physical meaning of this object? We think that this object is equivalent to the tensionless D0-branes discovered recently in [38, 39] in the framework of non-commutative theory. It was argued in [41] that these objects are gauge equivalent to the vacuum. The same problem was discussed in [21]. We see that we can obtain tensionless D0-brane in our approach. It would be very interesting to study fluctuation around this solution. We hope to return to this puzzle in the future.

4. D1-brane

In this section we will consider more general solution when $V(T)$ and $f(T)$ does not commute with Φ. Then the equation of motion for Φ^i has a form

$$\lambda\left[T,[\Phi^i,T]f(T)\right]+\lambda\left[T,f(T)[\Phi^i,T]\right]+\frac{\lambda^2}{2}\left[\Phi^k,[\Phi^i,\Phi^k]V(T)\right]+ \\ +\frac{\lambda^2}{2}\left[\Phi^k,V(T)[\Phi^i,\Phi^k]\right]=0\,, \tag{4.1}$$

and the equation of motion for tachyon

$$\frac{dV(T)}{dT}\left(1-\frac{\lambda^2}{4}[\Phi^i,\Phi^j][\Phi^i,\Phi^j]\right)-\lambda\frac{df(T)}{dT}[\Phi^i,T][\Phi^i,T]- \\ -\lambda\left(\left[[\Phi^i,T]f(T),\Phi^i\right]+\left[f(T)[\Phi^i,T],\Phi^i\right]\right)=0\,. \tag{4.2}$$

We take an ansatz

$$T=F(\hat{x}_1)=\sum b_n\hat{x}_1^n\,,\qquad \Phi^2=k^{-1}\hat{x}_2\,,\qquad [\hat{x}_1,\hat{x}_2]=ik\,, \\ \Phi^i=70\,,\qquad i=1,3,\dots,9\,, \tag{4.3}$$

where $F(x)$ approaches $T_{\min}$ for $x\to-\infty$ and $-T_{\min}$ for $x\to\infty$. Then

$$[\hat{x}_1^2,\hat{x}_2]=2ik\hat{x}_1\,,\qquad [\hat{x}_1^3,\hat{x}_2]=3ik\hat{x}_1^2\,,\qquad \dots\,,\qquad [\hat{x}_1^n,\hat{x}_2]=nik\hat{x}_1^{n-1}\,. \tag{4.4}$$

Using this result we obtain

$$[T,\Phi^2]=k^{-1}\left[\sum_n b_n\hat{x}_1^n,\hat{x}_2\right]=i\sum_n b_n\hat{x}_1^{n-1}n=i\frac{dT}{d\hat{x}_1}\,, \tag{4.5}$$

and consequently

$$[T^2,\Phi^2]=2iT\frac{dT}{d\hat{x}_1}\,,\qquad [T^4,\Phi^2]=4iT^3\frac{dT}{d\hat{x}_1}\,,\qquad \dots\,, \\ [T^{2n},\Phi^2]=i2nT^{2n-1}\frac{dT}{d\hat{x}_1}\,. \tag{4.6}$$

With these results in hand we obtain

$$\left[[\Phi^2,T]f(T),\Phi^2\right]=\frac{d}{d\hat{x}_1}(T'f(T))\,,\qquad \left[f(T)[\Phi^2,T],\Phi^2\right]=\frac{d}{d\hat{x}_1}(T'f(T))\,, \tag{4.7}$$

where $T' = dT/d\hat{x}_1$. Then the equation of motion for tachyon has a form

$$\frac{dV}{dT} - \lambda \frac{df}{dT} T'^2 - 2\lambda f T'' = 0 \,. \tag{4.8}$$

After multiplication with T' we get the result

$$V'(T(\hat{x}_1)) = (\lambda f T'^2)' \longrightarrow V(T) = \lambda f(T) T'^2 \,, \tag{4.9}$$

where the integration constant has been set to zero. In the previous expression we have used

$$(V(T(\hat{x}_1)))' = \sum a_n (T^{2n})' = \sum a_n 2n T^{2n-1} T' = \frac{dV}{dT} T' \,. \tag{4.10}$$

The equation of motion for Φ^2 is trivially satisfied since

$$\big[T, [\Phi^2, T] f(T)\big] = -i[T, T'V(T)] = 0 \,. \tag{4.11}$$

An energy of this solution is

$$E = \frac{C_0}{g_s} \operatorname{Str} \mathcal{V}(T) = 2\frac{C_0}{g_s} \operatorname{Tr} V(T(\hat{x}_1)) \,, \tag{4.12}$$

where we have used (4.9). Since we do not known the exact form of the $f(T)$ function we cannot determine the tachyon field so that we will work with (4.12) without exact form of the tachyon field $T = F(\hat{x}_1)$. Note that in this solution we do not need to presume that $T_{\min}$ is extreme of $f(T)$ with $f(T_{\min}) = 0$. It seams that this solution is more general one than the solution given in the section 3.

We must stress one important thing. We work in this section in the limit $N \to \infty$, when we can replace the matrices with the abstract operators action on Hilbert space, in the same way as in Matrix theory [46, 47, 48, 51]. Then $\hat{x}_1, \hat{x}_2$ are as the same operators as operators of coordinate and impuls for one particle system in standard quantum mechanics and we can easily calculate the trace in (4.12)

$$\begin{aligned} E &= \frac{C_0}{g_s} \operatorname{Tr} 2V(T(\hat{x}_1)) = 2\frac{C_0}{g_s} \int dx_2 \, \langle x_2 | \, V(T(\hat{x}_1)) \, | x_2 \rangle \\ &= \frac{2C_0}{g_s} \int dx_2 dx_1' dx_1'' \langle x_2 | x_1' \rangle V(T(x_1)) \langle x_1' | x_1'' \rangle \langle x_1'' | x_2 \rangle \\ &= 2\frac{C_0}{g_s} \int dx_2 dx_1 |\langle x_2 | x_1 \rangle|^2 V(T(x_1)) = \frac{2C_0}{g_s 2\pi k} \int dx_1 dx_2 V(T(x_1)) \,, \end{aligned} \tag{4.13}$$

where we have used the standard normalisation

$$\langle x_1 | x_2 \rangle = \frac{1}{\sqrt{2\pi k}} e^{i x_1 x_2 / k} \,, \tag{4.14}$$

where $|x_1\rangle \, , |x_2\rangle$ are eigenvectors of $\hat{x}_1$ and $\hat{x}_2$ with the normalisation condition $\langle x_1 | x_1' \rangle = \delta(x_1 - x_1'), \langle x_2 | x_2' \rangle = \delta(x_2 - x_2')$. In [14, 58] the energy of tachyon lump on unstable non-BPS D-brane was calculated. It was shown that the tension of resulting lump (in linearised approximation) is given with the integral $C_0 \int dx V(T(x)) \sim T_{-1} = 2\pi$. We cannot write equality since we do not know the precise form of $T = F(x)$ and we do not know the exact form o tachyon potential. However we can expect that this integral gives the result proportional to the tension of D(−1)-brane and then we get

$$E \sim \frac{2\pi}{g_s 4\pi^2\alpha'\tilde{k} g_s} \int dx_2 = \frac{T_1}{g_s \tilde{k}} \int dx_2 \,, \qquad \tilde{k} = k\lambda^{-1} \,, \tag{4.15}$$

which corresponds to the energy of D1-brane. The factor $\tilde{k}$ can be absorbed with coordinate redefinition. We claim that the energy of this configuration corresponds to the energy of D1-brane. This conclusion is also supported with the analysis of the WZ term (3.8)

$$\begin{aligned} I_{\mathrm{WZ}} &= \frac{\mu_{-1}}{2T_{\min}} \int dt \,\mathrm{Tr}\, i[\Phi^2, T] C^{(2)}_{2t} = \frac{\mu_{-1}}{2T_{\min}} \frac{1}{2\pi k} \int dt dx_1 dx_2 \frac{dT(x_1)}{dx_1} C^{(2)}_{2t} \\ &= \frac{\mu_{-1}}{2T_{\min}} \frac{1}{4\pi^2\alpha'\tilde{k}} \int dt dx_1 \left(T(\infty) - T(-\infty)\right) C^{(2)}_{2t} = \mu_1 \int dt dx_2 C^{(2)}_{t2} \,, \end{aligned} \tag{4.16}$$

where we have used $T(\infty) = -T_{\min}, T(\infty) = T_{\min}$, and have dropped the factor $\tilde{k}$. This is precisely the coupling between D1-brane and RR two form. It is remarkable that this exact result does not depend on the precise form of tachyon field. We hope that this result gives strong evidence that (4.3) really leads to the emergence of D1-brane. However, we must stress again that it seams to be hopeless to calculate exactly the energy of resulting configuration without knowledge of exact BI action for non-BPS D0-brane. On the other hand, recent results given in [44] suggest that higher derivative terms could be gauge artefacts only and then it seams to be possible to obtain exact solution.

4.1 Other BPS D-branes from non-BPS D0-branes

In this subsection we will see that we can obtain all BPS D-branes through tachyon condensation in the same way as a D1-brane. Let us consider an ansatz

$$\begin{array}{lll} T = F(\hat{x}_1) \,, & \Phi^1 = k^{-1}\hat{x}_2 \,, & [\hat{x}_1, \hat{x}_2] = ik \,, \\ \Phi^{2i} = k_i^{-1}\hat{p}_i \,, & \Phi^{2i+1} = k_i^{-1}\hat{q}_i \,, & [\hat{p}_i, \hat{q}_i] = ik_i \,, \qquad i = 1, \dots, p \,, \\ \Phi^i = 0 \,, & i = 2p+2, \dots, 9 \,. & \end{array} \tag{4.17}$$

It is easy to see that this ansatz solves (4.1) and from (4.2) we obtain

$$V(T(\hat{x}_1)) \left(1 + \frac{\lambda^2}{2} \sum_{i=1}^{p} \frac{1}{k_i}\right) = \lambda f(T(\hat{x}_1)) T'^2 \,. \tag{4.18}$$

We choose the Hilbert space basis

$$|\psi\rangle = |x_1\rangle \otimes |p_1\rangle \otimes \cdots \otimes |p_p\rangle\,, \qquad \langle\psi|\psi'\rangle = \delta(x_1 - x_1')\delta(p_1 - p_1') \cdots \delta(p_p - p_p')\,. \tag{4.19}$$

Then the energy of given configuration is equal to

$$\begin{aligned} E &= \frac{C_0}{g_s} 2\,\mathrm{Tr}\, V(T(x_1)) \left(1 + \frac{\lambda^2}{2}\sum_{i=1}^{p}\frac{1}{k_i^2}\right) \\ &\sim \frac{T_{2p+1}}{\tilde{k}\prod_{i=1}^{p}\tilde{k}_i}\left(1 + \frac{1}{2}\sum_{i=1}^{p}\frac{1}{\tilde{k}_i^2}\right)\int dx_2 dp_1 dq_1 \dots dp_p dq_p\,. \end{aligned} \tag{4.20}$$

This is proportional to the energy of D$(2p+1)$-brane since this energy scales as V_{2p+1} which is a volume on which this D-brane is wrapped. We have also defined $\tilde{k}_i = \lambda^{-1}k_i$. These factors can be dropped out from the action after redefinition $dp_i dq_i/\tilde{k}_i \rightarrow dx_{2i}dx_{2i+1}$. Since we used the action in linear approximation, the commutators should obey

$$\lambda^2[\Phi^i, \Phi^k]^2 \ll 1 \Longrightarrow \frac{\lambda}{k} \ll 1\,. \tag{4.21}$$

In limit $k \rightarrow \infty$ we can neglect the second term in the bracket and after the second redefinition $x_2 \rightarrow x_1$ we obtain the result

$$E \sim T_{2p+1}\int dx_1 dx_2 \dots dx_{2p+1}\,, \tag{4.22}$$

which suggests that the resulting configuration is D$(2p+1)$-brane. This claim is also supported with the analysis of the Wess-Zumino term which has a form

$$\begin{aligned} I_{\mathrm{WZ}} &= \frac{\mu_{-1}}{2T_{\mathrm{min}}}\int dt\,\mathrm{Str}\left(e^{i\lambda \mathbf{i}_\Phi \mathbf{i}_\Phi} i[\mathbf{i}_\Phi, T]\sum_n C^{(n)}\right) \\ &= \frac{\mu_{-1}}{2T_{\mathrm{min}}}\int dt\,\mathrm{Str}\, i[\Phi^1, T]C^{(2)}_{1t} - \\ &\quad - \frac{\mu_{-1}}{4T_{\mathrm{min}}}\sum_{i=1}^{p}\int dt\,\mathrm{Str}\,\lambda[\Phi^{2i}, \Phi^{2i+1}][\Phi^1, T]C^{(4)}_{1,2i+1,2i,t} - \\ &\quad - \frac{i\mu_{-1}}{8T_{\mathrm{min}}}\sum_{i=1, j\neq i}^{p}\int dt\,\mathrm{Str}\,\lambda^2[\Phi^{2i}, \Phi^{2i+1}][\Phi^{2j}, \Phi^{2j+1}][\Phi^1, T]C^{(6)}_{1,2j+1,2j,2i+1,2i,t} + \\ &\quad \cdots + \frac{i(i)^p\lambda^p\mu_{-1}}{2T_{\mathrm{min}}}\int dt\,\mathrm{Str}[\Phi^2, \Phi^3]\cdots[\Phi^{2p}, \Phi^{2p+1}][\Phi^1, T]C^{(2p+2)}_{1,2p+1,2p,\dots,3,2,t}\,. \end{aligned} \tag{4.23}$$

The first term in (4.23) corresponds to the coupling of D1-brane to two form $C^{(2)}$, as we have seen in (4.16). We will see that this configuration is charged with respect to $C^{(2)}, C^{(2)}, \ldots, C^{(2p)}$[2]. The same thing also arises in the construction of higher-dimensional objects in Matrix theory [46]–[51]. The second term in (4.23) gives

$$\sum_{i=1}^{p} \frac{\mu_1}{2\pi\lambda\tilde{k}_i} \int dt dx_1 dp_i dq_i C^{(4)}_{t1,2i,2i+1} = \sum_{i=1}^{p} \mu_3 \int dt dx_1 dx_{2i} dx_{2i+1} C^{(4)}_{t1,2i,2i+1}$$
$$= \sum_{i=1}^{p} \mu_3 \int_{M_i} C^{(4)} , \tag{4.24}$$

where M_i is a submanifold parameterised with t, x_1, x_{2i}, x_{2i+1}. It is clear that the previous term corresponds to p D3-branes wrapped submanifolds M_i. Again, their interpretation is the same as in Matrix theory.

In the same way we can proceed with other terms. For example, let us consider the third term in (4.23) with $i = 1,\ j = 2$. Then we obtain

$$-\frac{i\mu_{-1}\lambda^2}{2T_{\min}} \int dt \, \mathrm{Str}[\Phi^2, \Phi^3][\Phi^4, \Phi^5][\Phi^1, T] C^{(6)}_{15432t} =$$
$$= -\frac{\mu_5}{\tilde{k}_1^2 \tilde{k}_2^2 \tilde{k}} \int dt dx_2 dp_1 dq_1 dp_2 dq_2 C_{15432t}$$
$$= \mu_5 \int dt dx_1 dx_2 \ldots dx_5 C_{t1\ldots5} = \mu_6 \int_{M_{12}} C^{(6)} , \tag{4.25}$$

where M_{12} is a six-dimensional submanifold parameterised with coordinates $t, x_1, \ldots,$ x_5. Finally, the last term in (4.23) gives

$$\mu_{2p+1} \int dt dx_1 dx_2 \ldots dx_{2p} C^{(2p+2)}_{t12\ldots2p} = \mu_{2p+1} \int C^{(2p+2)} . \tag{4.26}$$

We see that this configuration really corresponds to the BPS D$(2p+1)$-brane. The fact that Φ^i in (4.17) do not commute suggests that the world-volume of resulting configuration is non commutative. It would be nice to study the fluctuation around this static solution. We hope to return to this interesting question in the future.

In the next section we would like to show that the action for N non-BPS D0-branes naturally leads to the non-commutative action for any higher-dimensional non-BPS D-brane, following [40].

5. Non-BPS D-branes from non-BPS D0-branes

We have seen in section (3) that the action for N non-BPS D0-branes has a trivial solution

$$T = T_{\min} 1_{N\times N} , \qquad \Phi^i = 0 , \qquad i = 1, \ldots, 9 , \tag{5.1}$$

[2]Since the various commutators are pure numbers we can replace the symmetrised trace with the ordinary one.

corresponding to the unstable vacuum with N non-BPS D0-branes. There is a question whether we can construct other non-BPS D-branes. Let us answer this question, following [40] and also earlier works [47], [67]–[70].

We start with the action

$$S = -\frac{C_0}{g_s}\int dt\,\mathrm{Str}\left(\sqrt{-\det(P[E_{tt}+E_{tI}(X^{IJ}-\delta^{IJ})E_{Jt}]}\right)\times \\ \times\sqrt{\det(Q^I_J)}\times F(T,\dot{T},\Phi^I,\dots)\,, \tag{5.2}$$

with

$$F = V(T)-\sum_{n=1}^{\infty}f_n(T)\lambda^n\left((E^{tt}-E^{tI}E_{IJ}E^{Jt})\dot{T}\dot{T}+iE^{tK}E_{KJ}[\Phi^J,T]\dot{T}- \\ -E_{IJ}[\Phi^I,T][\Phi^J,T]\right)^n. \tag{5.3}$$

We introduce the constant background metric $E_{IJ}=g_{IJ}$, with vanishing B_{IJ} and with $E^{tI}=0$. Then $Q^{IJ}=g^{IJ}+i\lambda[\Phi^I,\Phi^J]$. We also use the notation $I,J,K,\dots=1,\dots,9$; $i,j,k,\dots=1,\dots,2p$; $a,b,c,\dots=2p+1,\dots,9$. We also assume that this background metric is block-diagonal with the blocks g_{ij},g_{ab} with $g_{ia}=0$. Let us propose an ansatz

$$T=0\,,\qquad \Phi^i_{\text{clas}}=\lambda^{-1}x^i\,,\qquad i=1,\dots,2p\,,\qquad [x^i,x^j]=i\Theta^{ij} \tag{5.4}$$

and other fields Φ^a, $a=2p+1,\dots,9$ in the form $\Phi^a=x^a1_{N\times N}$, which describe the transverse positions of the resulting D-brane. It is easy to see that this ansatz (5.4) is a solution of the equation of motion of the whole action since the commutators between tachyon and scalar field vanish and also from the fact that commutators of two $\Phi's$ are pure numbers and consequently $[\Phi^i,[\Phi^j,\Phi^i]]=0$.

Next we will analyse the fluctuation around this background. We will closely follow [40] and write

$$C_i=\lambda B_{ij}\Phi^j=\lambda B_{ij}\Phi^j_{\text{clas}}+\lambda\Phi^j_{\text{fluct}}=B_{ij}x^j+\hat{A}_i\,,\qquad i=1,\dots,2p\,, \\ \Phi^a=\Phi^a\,,\qquad a=2p+1,\dots,9\,,\qquad T_{\text{fluct}}=T\,, \tag{5.5}$$

and calculate

$$[C_i,C_j]=-iB_{ij}+B_{ik}[x^k,\hat{A}_j]-B_{jl}[x^l,\hat{A}_i]+[\hat{A}_i,\hat{A}_j]\,, \\ [C_i,\Phi^a]=B_{ik}[x^k,\Phi^a]+[\hat{A}_i,\Phi^a]\,, \tag{5.6}$$

where we have used

$$B_{ik}B_{jl}[x^k,x^l]=-B_{ik}i(B^{-1})^{kl}B_{lj}=-iB_{ij}\,. \tag{5.7}$$

Using

$$\Phi^i=\lambda^{-1}\Theta^{ik}C_k\,, \tag{5.8}$$

we obtain

$$i\lambda[\Phi^i,\Phi^j]=\lambda^{-1}\Theta^{ik}(\hat{F}_{kl}-B_{kl})\Theta^{lj}\,, \tag{5.9}$$

where

$$\hat{F}_{kl}=-iB_{ik}[x^k,\hat{A}_l]+iB_{jl}[x^l,\hat{A}_i]-i[\hat{A}_i,\hat{A}_j]\,. \tag{5.10}$$

The best thing how we can study the fluctuation around the classical solution is to start with the original form of the determinant

$$D=\det\begin{pmatrix} g_{tt} & \lambda D_t\Phi^J \\ -\lambda D_t\Phi^I & g^{IJ}+i\lambda[\Phi^I,\Phi^J]\end{pmatrix}=\det\begin{pmatrix} D_{tt} & D_{tj} & D_{tb} \\ D_{it} & D_{ij} & D_{ib} \\ D_{at} & D_{aj} & D_{ab}\end{pmatrix}, \tag{5.11}$$

with the action in the form

$$S=-\frac{C_0}{g_s}\int dt\,\mathrm{Str}\,\sqrt{\det g_{IJ}}\sqrt{-\det D}\times F(T,\Phi,\dots)\,, \tag{5.12}$$

where the factor $\sqrt{\det g_{IJ}}$ arises from T-duality transformation of the dilation (2.7).

We have also written D_t instead ∂_t in order to obtain more symmetric expression. (Remember, we are working in gauge $A_0=0$.) Then we obtain

$$\begin{aligned}
&D_{tb}=\lambda D_t\Phi^b\,, \qquad D_{it}=-\Theta^{ik}D_t\hat{A}_k\,, \qquad D_{tj}=-D_t\hat{A}_k\Theta^{kj}\,, \qquad D_{at}=-\lambda D_t\Phi^a\,,\\
&D_{ij}=g^{ij}+i\lambda[\Phi^i,\Phi^j]=g^{ij}+\lambda^{-1}\Theta^{ik}(\hat{F}_{kl}-B_{kl})\Theta^{lj}\,,\\
&D_{ib}=-\Theta^{ik}D_k\Phi^b\,, \qquad D_{aj}=-D_k\Phi^a\Theta^{kj}\,,
\end{aligned} \tag{5.13}$$

where

$$iD_kM=[C_k,M]=B_{kl}[x^l,M]+[\hat{A}_l,M]\,. \tag{5.14}$$

Finally we have

$$D_{ab}=g^{ab}+i\lambda[\Phi^a,\Phi^b]=Q^{ab}\,. \tag{5.15}$$

Then

$$D=\det\begin{pmatrix} D_{tt}-D_{tb}(Q^{-1})_{bc}D_{ct} & D_{tj}-D_{tb}(Q^{-1})_{bc}D_{cj} & D_{tb} \\ D_{it}-D_{ib}(Q^{-1})_{bc}D_{ct} & D_{ij}-D_{ib}(Q^{-1})_{bc}D_{cj} & D_{ib} \\ 0 & 0 & Q^{ab}\end{pmatrix}. \tag{5.16}$$

The first block-diagonal term suggests emergence of D($2p$)-brane. We will show this more precisely

$$D_{tt}-D_{tb}(Q^{-1})_{bc}D_{ct}=g_{tt}+\lambda^2D_t\Phi^a(Q^{-1})_{ab}D_t\Phi^b=P[g_{tt}+g_{ta}(X^{ab}-\delta^{ab})g_{bt}]\,, \tag{5.17}$$

where meaning of various terms it the same as in the section 3. In the similar way we obtain

$$\begin{aligned}
D_{tj}-D_{tb}(Q^{-1})_{bc}D_{cj}&=(-D_t\hat{A}_k+\lambda D_t\Phi^b(Q^{-1})_{bc}D_k\Phi^c)\Theta^{kj}\\
&=(-\lambda\hat{F}_{tk}+P[g_{tk}+g_{ta}(X^{ab}-\delta^{ab})g_{bk}])\lambda^{-1}\Theta^{kj}\,,
\end{aligned}$$

$$
\begin{aligned}
D_{it} &= -\lambda^{-1}\Theta^{ik}(-\lambda\hat{F}_{kt} + P[g_{kt} + g_{ka}(X^{ab} - \delta^{ab})g_{bt}])\,, \\
D_{ij} - D_{ib}(Q^{-1})_{bc}D_{cj} &= g^{ij} - \lambda^{-1}(\Theta B\Theta)^{ij} + \lambda^{-1}(\Theta\hat{F}\Theta)^{ij} - \Theta^{ik}D_k\Phi^b(Q^{-1})_{bc}D_b\Phi^c\Theta^{kj} \\
&= -\Theta^{ik}\lambda^{-1}(B_{kl} - \hat{F}_{kl} + P[G_{kl} + G_{ka}(X^{ab} - \delta^{ab})G_{bl}])\Theta^{lj}\,; \\
G_{ij} &= -\lambda^2\Theta_{ik}g^{kl}\Theta_{lj}\,.
\end{aligned}
\tag{5.18}
$$

We combine D_{tt} with D_{ij}, D_{it}, D_{ij} into one single matrix $\mathcal{D}_{ij}, i,j = 0,1,\ldots,2p$. Then D is equal to

$$
D = (\det\lambda\Theta)^2\det\mathcal{D}'\det Q^{ij}\,, \qquad \mathcal{D} = \Theta\mathcal{D}'\Theta\,, \tag{5.19}
$$

where we have used

$$
\mathcal{D} = \begin{pmatrix} A & BX \\ YC & -YPX \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -Y \end{pmatrix}\begin{pmatrix} A & B \\ -C & P \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & X \end{pmatrix}. \tag{5.20}
$$

In previous expression $X = Y = \lambda\Theta$. We can also write

$$
\det g\det Q^{ab} = \det(Q^a_b)\,, \tag{5.21}
$$

where we have used the fact that the original action (5.12) contains the factor $\sqrt{\det(g_{IJ})} = \sqrt{\det(g_{ij})}\sqrt{\det(g_{ab})}$. We can analyse F function (5.3) in the similar way. More precisely

$$
-\lambda^{-1}\Theta^{ik}\Theta^{jl}[C_k, T][C_l, T] = \lambda^{-1}\Theta^{ik}\Theta^{jl}D_kTD_lT = -\lambda^{-1}(\Theta DTDT\Theta)^{ij}\,, \tag{5.22}
$$

and consequently

$$
\begin{aligned}
-g_{ij}[\Phi^i, T][\Phi^j, T] &= -g_{ij}\lambda^2[\Theta^{ik}C_k, T][\Theta^{jl}C_l, T] \\
&= -G^{kl}[C_k, T][C_l, T] = G^{ij}D_iTD_jT\,.
\end{aligned}
\tag{5.23}
$$

As a result we obtain

$$
F = V(T) - \sum_{n=1} f_n(T)\lambda^n\left(G^{ij}D_iTD_jT - g_{ab}[\Phi^a, T][\Phi^b, T]\right)^n\,, \tag{5.24}
$$

where we have included g_{tt} into the definition of G_{ij}.

With these results in hand we get the final result

$$
\begin{aligned}
S = &-\frac{C_0}{g_s}\,\mathrm{Str}\int dt\sqrt{\det(g_{ij})}\det(\lambda^{-1}\Theta)\,\times \\
&\times\sqrt{-\det(\lambda(\hat{F}_{kl} - B_{kl}) + P[G_{kl} + G_{ka}(X^{ab} - \delta^{ab})G_{bl}]}\,\times \\
&\times\sqrt{\det(Q^a_b)}F(T, DT, \ldots)\,,
\end{aligned}
\tag{5.25}
$$

where we have used

$$
\det(A+B) = \det(A+B)^T = \det(A-B)\,, \qquad A^T = A\,, \qquad B^T = -B\,. \tag{5.26}
$$

In the previous equation the trace goes over $N \times N$ matrices. Following [40], we can take the limit $N \to \infty$. Then there is a standard relation between the trace over Hilbert space and integration in non-commutative theory, see [37, 40]

$$\int d^{2p}x = (2\pi)^n \sqrt{\det \Theta}\, \mathrm{Tr}\ . \tag{5.27}$$

We must also remember that the multiplication in the resulting action is non-commutative one with the ordinary product replaced with the star product since, as we can see from (5.4), the world-volume of a non-BPS D($2p$)-brane is non commutative. With using

$$G_s = g_s \sqrt{\frac{\det \lambda B}{\det g}}\,, \qquad \sqrt{\det \lambda^{-1}\Theta} = \lambda^{-p}\sqrt{\det \Theta}\,, \tag{5.28}$$

we obtain from (5.25) the action for non-BPS D($2p$)-brane

$$\begin{aligned} S = -\frac{C_{2p}}{G_s}\int dt d^{2p}x \sqrt{-\det\left(\lambda(\hat{F}_{kl} - B_{kl}) + P[G_{kl} + G_{ka}(X^{ab} - \delta^{ab})G_{bl}]\right)} \times \\ \times \sqrt{\det(Q^a_b)}\left(V(T) - \sum_{n=1} f_n(T)\lambda^n \left(G^{ab}D_aTD_bT - g_{ij}[\Phi^i, T][\Phi^j, T]\right)^n\right), \end{aligned} \tag{5.29}$$

where we have used

$$\frac{C_0}{(2\pi\lambda)^p} = C_{2p}\,. \tag{5.30}$$

We have seen that N non-BPS D-branes in the limit $N \to \infty$ have solution corresponding to non-BPS D-branes of higher dimension. This solution is in some sense dual to the tachyon condensation on the world-volume of space-time filling branes with non-commutative world-volume. In fact, there is a closed relation between non-BPS D0-branes and action for non-BPS D-brane written in operator formalism [37, 39, 44]. In this section we have demonstrated this relation more explicitly. The generalisation to the case of non-abelian non-BPS D($2p$)-brane is straightforward [40].

6. Conclusion

In this short note we have presented some results considering tachyon condensation in the system of N non-BPS D0-branes. We have seen in the section 2 that there is a solution with zero mass which we have interpreted as a tensionless D0-brane [39, 38]. This problem is similar to the problem of tensionless circular D8-brane in [21]. However, there is a puzzle. If this was genuine light state in type-II string theory they should have been known already from other studies. At present we do not know how resolve this puzzle. Resolution of this problem has been suggested in [41] in the framework of non-commutative geometry which was based on the extra $\mathbb{Z}_2$ discrete gauge symmetry of the action for non-BPS D-brane. We hope to return to this important question in the future.

In section 4 we have proposed an ansatz which leads to the emergence of BPS D-branes from non-BPS D0-branes. Unfortunately, we were not able to calculate the tension of resolution object directly, since we have used the linear approximation only. However, from the form of the energy of this configuration which scales as a energy of BPS D-brane and also from the charge of resulting configuration we can claim the these solutions really describe BPS D-branes in type-IIB theory since non-BPS D0-branes are present in type-IIB theory. It would be nice to go beyond linear approximation which seams to be impossible at present since we do not know the exact form of the action. On the other hand, it was suggested in [44] that it is possible that higher order terms in the action are only gauge artefacts. It would be nice to have some exact proof this intriguing conjecture.

We have also seen that from the action for non-BPS D0-branes we can obtain action for higher-dimensional non-BPS D($2p$)-branes in the very elegant way used in the construction of higher dimensional branes in Matrix theory and in type-IIB theory. We have seen that this analysis is valid for the whole effective action without restriction to the linear approximation. The same analysis could be possible with the Wess-Zumino term for a non-BPS D0-branes which could lead to the Wess-Zumino term for non-commutative D-branes presented recently in the beautiful paper [45]. We hope to return to this question in the future.

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